Accelerated Math 3
Writing

Name Dr. Gammill

1. Describe what causes holes in rational functions and given an example.

When there is a common factor in the num + denom.

$f(x) = \frac{(5x+1)(x+1)}{(2x-1)(x+1)}$

2. Where do the zeros of a rational function come from? Give an example.

By setting the num = 0 + solving. $f(x) = \frac{2x+6}{x}$

Zero: $2x+6=0 \quad 2x=-6 \quad x=-3$

3. Where do the vertical asymptotes of a rational function come from? Give an example of a rational function that has no vertical asymptote.

By setting the denom = 0.
(Because that's where the function is undefined)

$y = \frac{5}{x+1}$ $x+1=0$ $x=-1$

4. Can a rational function have both horizontal and slant asymptotes?

Nope

5. When do slant asymptotes occur? Give an example.

When the rational function is top-heavy (err. the deg. of the num > the deg of the denom)

$f(x) = \frac{x^3}{x^2+1}$

Write a rational function given the following information:

6. Resembles $y = x-4$ with holes at 7 and -1.

$f(x) = \frac{(x-4)(x-7)(x+1)}{(x-7)(x+1)}$

7. Has a $\boxed{\text{zero at 0}}$, $\boxed{\text{holes at 7 and }-2}$, vertical asymptote at $\boxed{x=7}$, and horizontal asymptote at $y=1$.

$f(x) = \frac{x(x-7)(x+2)}{(x-7)(x+2)(x-7)}$

8. Has no zero, vertical asymptote at $x = 3\frac{1}{2}$, (4, -1) is on the graph.

x f(x)

$x = 3\frac{1}{2} = \frac{7}{2}$

$x = \frac{7}{2}$

$2x = 7$

$2x-7=0$

$f(x) = \frac{a}{2x-7}$

$-1 = \frac{a}{2(4)-7}$

$\rightarrow -1 = \frac{a}{1} \rightarrow a = -1$

$\boxed{f(x) = \frac{-1}{2x-7}}$

9. Has a zero at 0, VA at $x = \frac{2}{3}$, HA at $y = \frac{2}{3}$

$f(x) = \frac{2x}{3x-2}$

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Rational Functions, part deux

1) Review: $f(x) = \dfrac{2x-5}{x+3}$ will have

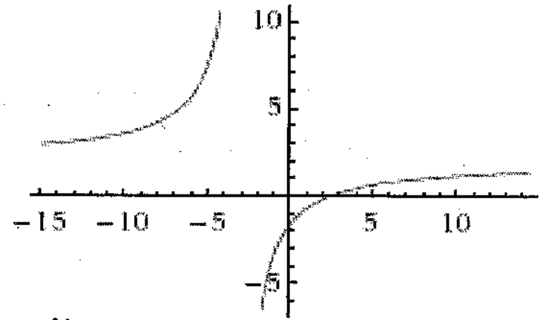
a) a vertical asymptote at $x=-3$,

b) a horizontal asymptote at $y=2$,

c) an x-intercept at $(5/2, 0)$

d) and a y-intercept at $(0, -5/3)$

Use this information to sketch f(x) to the right.



Use long division to divide 2x − 5 by x + 3 below.

$$x+3 \overline{\big)\, 2x-5} \quad \text{quotient: } 2 + \frac{-11}{x+3}$$
$$\underline{2x+6}$$
$$-11$$

So, $\dfrac{2x-5}{x+3} = \dfrac{-11}{x+3} + 2$

2) How does this long division explain the transformations that have happened to y = 1/x to produce the graph of f(x)?

For $f(x) = \dfrac{-11}{x+3} + 2$, $a=-11$, $h=-3$, $k=2$

$a=-11 \rightarrow$ Stretch of 11, refl. over x-axis

$h=-3 \rightarrow$ Hor. shift 3 left $\rightarrow x=-3$ is the VA

$k=2 \rightarrow$ Vert shift up 2 $\rightarrow y=2$ is the HA

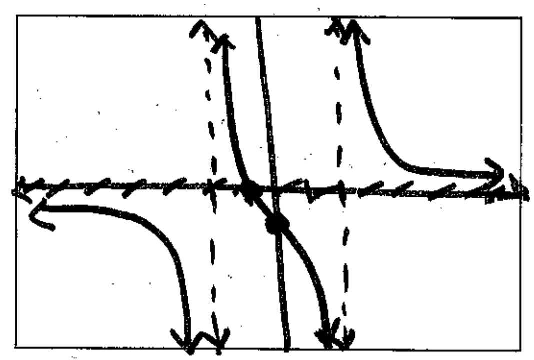
This agrees w/ #1.

3) $g(x) = \dfrac{4x+4}{x^2-4}$ has vertical asymptotes at $x = \pm 2$,

a horizontal asymptote at $y=0$,

a y-intercept at $(0,-1)$, an x-intercept at $(-1,0)$

Use this information to sketch this graph to the right.



$\rightarrow f(x) = \dfrac{x(x+1)(x-1)}{(x-2)(x+1)} = \dfrac{x(x-1)}{x-2} = \dfrac{x^2-x}{x-2}$ hole@ $(-1, 1/3)$

4) $f(x) = \dfrac{x^3-x}{x^2-x-2}$ will have a hole at $(-1, 1/3)$ a vertical asymptote at $x=2$, x-intercepts at

a) $(0,0)$ $(1,0)$, a y-intercept at $(0,0)$ and no horizontal asymptote. Why?

The fraction is top-heavy; therefore, the function has a slant asymptote. $\boxed{\text{SA: } y=x+1}$

$$x-2 \overline{\big)\, x^2 - x + 0} \quad \text{quotient: } x + 1 + \frac{2}{x-2}$$
$$\underline{-x^2+2x} \downarrow$$
$$x$$
$$\underline{+x+2}$$
$$2$$

8) Find the slant asymptotes of these three functions:

A) $f(x) = \dfrac{3x^2+5x-7}{x+3}$

$$\begin{array}{r}
3x-4+5/x+3 \\
x+3\,\overline{)\,3x^2+5x-7} \\
3x^2+9x \\
\hline
-4x-7 \\
-4x-12 \\
\hline
5
\end{array}$$

SA: $y=3x-4$

B) $y = \dfrac{x^3+6x^2-2x+7}{x^2+2x-4}$

$$\begin{array}{r}
x+4+\frac{-6x+23}{} \\
x^2+2x-4\,\overline{)\,x^3+6x^2-2x+7} \\
x^3+2x^2-4x \downarrow \\
\hline
+4x^2+2x+7 \\
+4x^2+8x-16 \\
\hline
-6x+23
\end{array}$$

Slant Asym: $y=x+4$

C) $g(x) = \dfrac{x^3+4x^2-5x}{x-2}$

$$\begin{array}{r}
x^2+6x+\frac{7x}{} \\
x-2\,\overline{)\,x^3+4x^2-5x} \\
x^3-2x^2 \\
\hline
6x^2-5x \\
6x^2-12x \\
\hline
7x
\end{array}$$

SA: $y=x^2+6x$

9) a) As you can see from example C), sometimes "slant asymptotes" aren't lines. What type of function is the asymptote in part C?

It's a quadratic function.

b) How did the equation of the original rational function predict this result?

Because the degree of the numerator is TWO more than the degree of the denominator.

c) What conclusions can you draw from this exercise?

After doing long division, the function has asymptotic behavior, approaching the quotient. The remainder has a negligible affect on the end behavior.