

HAA - Test Review

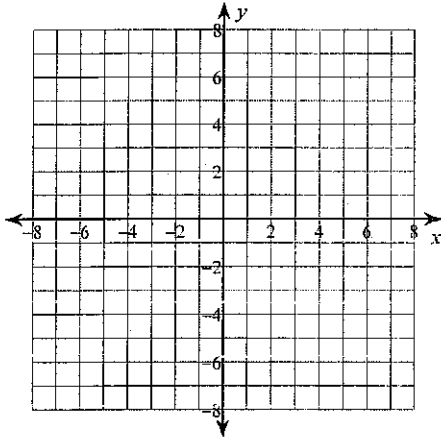
Name _____

Graphing Rational Functions

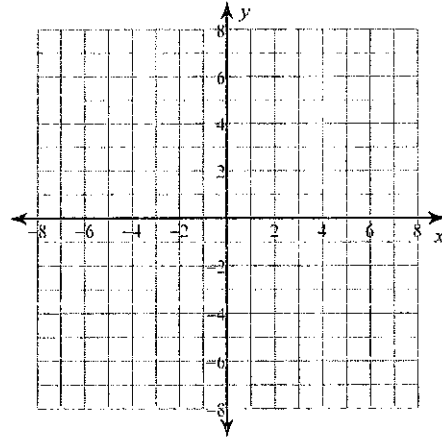
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Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.

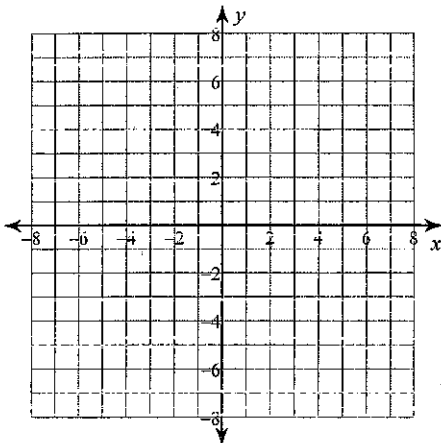
1) $f(x) = \frac{x^2 - 2x - 3}{2x^2 - 8}$



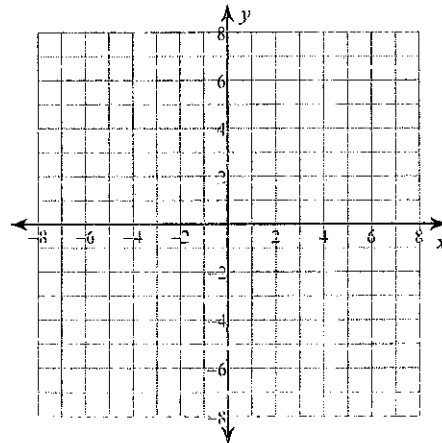
2) $f(x) = \frac{x^2 - 4x + 3}{-4x}$



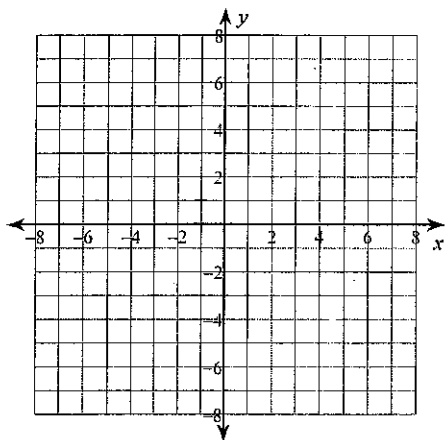
3) $f(x) = \frac{x^2 + x - 2}{-4x - 16}$



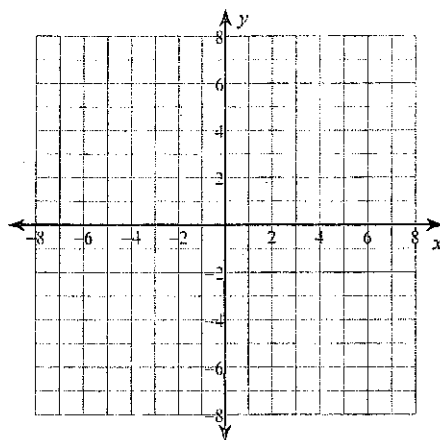
4) $f(x) = \frac{x^3 - 6x^2 + 8x}{3x^2 - 21x + 36}$



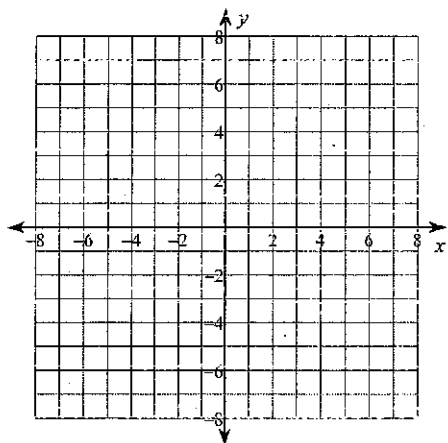
$$5) f(x) = \frac{x^2 - 3x + 2}{-3x^2 - 9x}$$



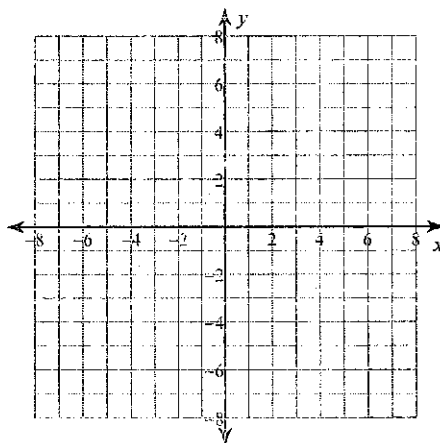
$$6) f(x) = \frac{x^3 - 16x}{4x^2 + 12x}$$



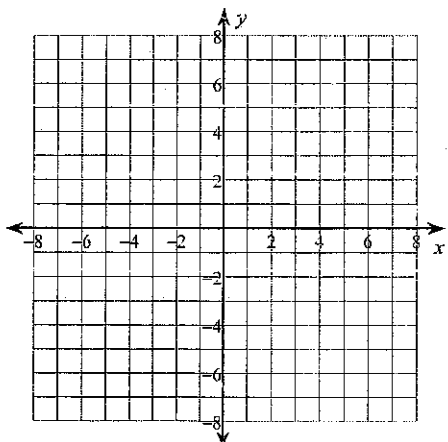
$$7) f(x) = \frac{x^2 + 3x - 4}{-3x - 6}$$



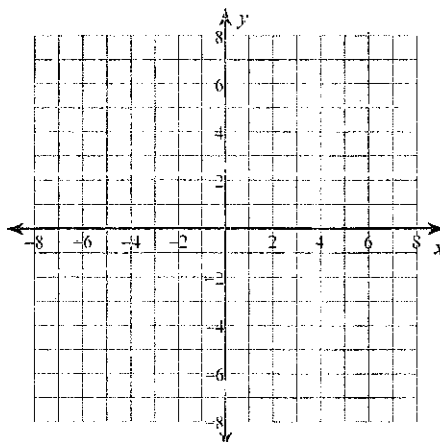
$$8) f(x) = \frac{x^2 - 2x}{-4x - 4}$$



$$9) f(x) = \frac{x^3 - 4x}{-3x^2 + 3x + 18}$$



$$10) f(x) = \frac{x^3 - 2x^2 - 3x}{-4x^2 + 16x}$$



HAA - Test Review Key

$$\textcircled{1} f(x) = \frac{x^2 - 2x - 3}{2x^2 - 8} = \frac{(x-3)(x+1)}{2(x+1)(x-1)} = \frac{x-3}{2(x-1)} = \frac{x-3}{2x-2} \quad \boxed{\text{HOLE @ } (-1, 1)}$$

<u>Denom</u> VA: $x=1$	<u>Num</u> $x-3=0$ $x=3$	<u>Ratios</u> yint: $(0, 3/2)$	<u>Range:</u> $(-\infty, 1/2) \cup (1/2, 1) \cup (1, \infty)$
<u>Domain:</u> $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$	<u>xint:</u> $(3, 0)$	<u>HA:</u> $y=1/2$	<u>EB</u> $x \rightarrow \pm \infty, f(x) \rightarrow 1/2$

$$\textcircled{2} f(x) = \frac{x^2 - 4x + 3}{-4x} = \frac{(x-3)(x-1)}{-4x} \quad -4x \overline{) \begin{array}{r} x^2 - 4x + 3 \\ \underline{x^2} \\ 0 - 4x + 3 \\ \underline{-4x} \\ 0 + 3 \end{array}}$$

<u>Denom</u> $-4x=0$ $x=0$ VA: $x=0$ D: $(-\infty, 0) \cup (0, \infty)$	<u>Num</u> $x=3$ $x=1$ xint: $(3, 0), (1, 0)$	<u>Ratios</u> yint: none SA: $y = -1/4x + 1$	<u>Extrema</u> $(-1.7, 1.9)$ (min) $(1.7, .1)$ max
		<u>Range:</u> $(-\infty, 0) \cup (1.9, \infty)$	

$$\textcircled{3} f(x) = \frac{x^2 + x - 2}{-4x - 16} = \frac{(x+2)(x-1)}{-4(x+4)} \quad -4x-16 \overline{) \begin{array}{r} x^2 + x - 2 \\ \underline{x^2 + 4x} \\ -3x - 2 \\ \underline{-3x - 12} \\ 10 \end{array}} \quad \cdot \frac{3x}{-4x}$$

<u>Denom</u> VA: $x=-4$ Domain $(-\infty, -4) \cup (-4, \infty)$	<u>Num</u> xint: $(-2, 0), (1, 0)$	<u>Ratios</u> yint: $(0, 1/8)$ SA: $y = -1/4x + 3/4$	<u>Extrema:</u> $(-7.1, 3.3)$ min $(-9, .2)$ max
		<u>Range:</u> $(-\infty, .2) \cup (3.3, \infty)$	

$$\textcircled{4} f(x) = \frac{x^3 - 6x^2 + 8x}{3x^2 - 21x + 36} = \frac{x(x-4)(x-2)}{3(x-3)(x-4)} = \frac{x(x-2)}{3(x-3)} = \frac{x^2 - 2x}{3x - 9}$$

<u>Hole @</u> $(4, 8/3)$	<u>Denom</u> $3x-9=0$ $3x=9$ VA $x=3$ D: $(-\infty, 3) \cup (3, 4) \cup (4, \infty)$	<u>Num:</u> $x^2 - 2x = 0$ $x(x-2) = 0$ $x=0, x=2$ xint: $(0, 0), (2, 0)$	<u>Ratios</u> yint: $(0, 0)$ SA: $y = 1/3x + 1/3$
$3x-9 \overline{) \begin{array}{r} x^2 - 2x \\ \underline{x^2 - 3x} \\ x \\ \underline{x-3} \\ 0 \end{array}}$			

$$\textcircled{5} \quad f(x) = \frac{x^2 - 3x + 2}{-3x^2 - 9x} = \frac{(x-2)(x-1)}{-3x(x+3)}$$

Denom

$$\begin{array}{l} x=0 \\ x=-3 \end{array} \left. \vphantom{\begin{array}{l} x=0 \\ x=-3 \end{array}} \right\} \text{VA}$$

$$D: (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$$

Num

$$x-2=0$$

$$x=2$$

$$x-1=0$$

$$x=1$$

x int:

$$(2, 0) \quad (1, 0)$$

Ratios

y int: none

Balanced degrees

$$\text{HA: } y = -\frac{1}{3}$$

$$\text{Range: } (-\infty, 0) \cup [0.9, \infty)$$

$$\searrow: (-\infty, -3) \cup (-3, 0.9)$$

$$\nearrow: (-0.7, 0) \cup (0, \infty)$$

* Relative Min *

$$(-0.7, 0.9)$$

$$⑥ f(x) = \frac{(x+4)(x-4)}{x+3} = \frac{x^2-16}{x+3}$$

$$x+3 \overline{) \begin{array}{r} x^2 + 0x - 16 \\ x^2 + 3x \\ \hline -3x - 16 \\ -3x - 9 \\ \hline -7 \end{array}}$$

Denom

$$x+3=0$$

$$VA: x=-3$$

Domain

$$(-\infty, -3) \cup (-3, 0) \cup (0, \infty)$$

Num

$$x \text{ int:}$$

$$(\pm 4, 0)$$

Ratios

$$y \text{ int: } (0, -\frac{16}{3})$$

$$SA: y = x-3$$

No extrema

$$\text{Range: } (-\infty, \infty)$$

$$SA: y = x-3$$

$$⑦ f(x) = \frac{x^2+3x-4}{-3x-6} = \frac{(x+4)(x-1)}{-3(x+2)} \quad -3x+6 \overline{) \begin{array}{r} x^2 + 3x - 4 \\ x^2 - 2x \\ \hline 5x - 4 \\ 5x - 10 \\ \hline 6 \end{array}}$$

Denom

$$-3(x+2)=0$$

$$VA: x=-2$$

Domain

$$(-\infty, -2) \cup (-2, \infty)$$

Num

$$x \text{ int:}$$

$$(-4, 0)$$

$$(1, 0)$$

Ratios

$$y \text{ int: } (0, \frac{2}{3})$$

$$SA: y = -\frac{1}{3}x - \frac{5}{3}$$

$$\text{Range: } (-\infty, \infty)$$

(No extrema)

$$SA: y = -\frac{1}{3}x - \frac{5}{3}$$

$$⑧ f(x) = \frac{x^2-2x}{-4x-4} = \frac{x(x-2)}{-4(x+1)} \quad -\frac{1}{4}x + \frac{3}{4} + \frac{3}{-4x-4} \quad \overline{) \begin{array}{r} x^2 - 2x + 0 \\ x^2 + x \\ \hline -3x \\ -3x - 3 \\ \hline 3 \end{array}}$$

Denom

$$VA: x=-1$$

~~Do~~

Domain

$$(-\infty, -1) \cup (-1, \infty)$$

Num

$$x=0$$

$$x=2$$

x int:

$$(0, 0)$$

$$(2, 0)$$

Ratios

$$y \text{ int: } (0, 0)$$

$$SA: y = -\frac{1}{4}x + \frac{3}{4}$$

Range:

$$(-\infty, -1] \cup [1.9, \infty)$$

Extrema (relative)

$$(-2.7, 1.9) \text{ min}$$

$$(1.7, 1) \text{ max}$$

$$9) f(x) = \frac{x^3 - 4x}{-3x^2 + 3x + 18} = \frac{x(x+2)(x-2)}{-3(x-3)(x+2)} = \frac{x(x-2)}{-3(x-3)} = \frac{x^2 - 2x}{-3x + 9}$$

Hole: $(-2, \frac{8}{15})$

$$\begin{array}{r} -\frac{1}{3}x - \frac{1}{3} \\ -3x + 9 \overline{) x^2 - 2x + 0} \\ \underline{x^2 - 3x} \\ x + 0 \\ \underline{x - 3} \\ 3 \end{array}$$

SA: $y = -\frac{1}{3}x - \frac{1}{3}$

Denom

$x = 3$ VA

Domain:

$(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

Num

$x = 0$

$x = 2$

x int:

$(0, 0)$

$(2, 0)$

Ratios

y int: $(0, 0)$

SA: $y = -\frac{1}{3}x - \frac{1}{3}$

Extrema

$(1.3, -2.2)$

$(4.7, -2.5)$

Range: $(-\infty, -2.5] \cup$

$[-2, \infty)$

$$10) f(x) = \frac{x^3 - 2x^2 - 3x}{-4x^2 + 16x} = \frac{x(x-3)(x+1)}{-4x(x-4)} = \frac{x(x-3)(x+1)}{-4x(x-4)}$$

$$= \frac{x(x-3)(x+1)}{-4x(x-4)}$$

Hole: $(0, -\frac{3}{16})$ SA: $y = -\frac{1}{4}x - \frac{5}{2}$

Denom

$x = 4$ VA

Domain

$(-\infty, 0) \cup (0, 4) \cup (4, \infty)$

Num

x int

$(3, 0)$

$(-1, 0)$

Ratios

y int:

$(0, -\frac{3}{16})$

(But this is the hole!)

SA: $y = -\frac{1}{4}x - \frac{1}{2}$

Range:

$(-\infty, -2.6] \cup [-4, \infty)$

Extrema: $(1.8, -4)$
 $(6.2, -2.6)$