Inverse Functions

The INVERSE of a function swaps the domain (x) and range. You can do this with a table of values, a graph, or a single point.

To find the inverse of a function algebraically:

1) Change the f(x) to y

2) Switch the x and the y.

3) Solve for y.

4) Replace y with $f^{-1}(x)$

★ $f^{-1}(x)$ means "f inverse" NOT $\frac{1}{f(x)}$

1.) $f(x) = 3x - 7$

$y = 3x - 7$

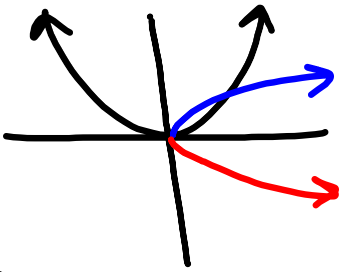
$x = 3y - 7$

$x + 7 = 3\boxed{y}$

$y = \frac{x+7}{3}$

$f^{-1}(x) = \frac{x+7}{3}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$



2.) $f(x) = 4x^2$

$y = 4x^2$

$x = 4y^2$

$\frac{x}{4} = y^2$

$y = \pm\sqrt{\frac{x}{4}} = \frac{\sqrt{x}}{\sqrt{4}}$

$y = \pm\frac{\sqrt{x}}{2}$

$f^{-1}(x) = \frac{\pm\sqrt{x}}{2}$

$y = a(x-h)^3 + k$

3.) $f(x) = x^3 + 4$

$y = x^3 + 4$

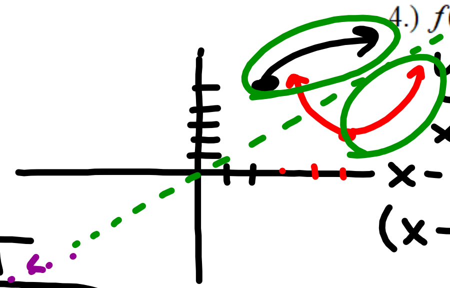
$x = y^3 + 4$

$x - 4 = y^3$

$y = \sqrt[3]{x-4}$

$f^{-1}(x) = \sqrt[3]{x-4}$

$y = a\sqrt[3]{x-h} + k$



$f(x) = a\sqrt{x-h} + k$

4.) $f(x) = \sqrt{x-2} + 5$

$y = \sqrt{x-2} + 5$

$x = \sqrt{y-2} + 5$

$x - 5 = \sqrt{y-2}$

$(x-5)^2 = y - 2$

$y = (x-5)^2 + 2$

$f^{-1}(x) = (x-5)^2 + 2$

• $y = a(x-h)^2 + k$

$y = ab^{x-h} + k$

5.) $y = 3^x$

$x = 3^y$

$y = \log_3 x$

$y^{-1} = \log_3 x$

$f^{-1}(x) = \log_3 x$

6.) $y = 2^{x-1}$

$x = 2^{y-1}$

$y - 1 = \log_2 x$

$y = \log_2 x + 1$

$y = a\log_b(x-h) + k$

7.) $y = \log_3 x$

$x = \log_3 y$

$y = 3^x$

$y^{-1} = 3^x$

$f^{-1}(x) = 3^x$

8.) $y = \log_4(x+2)$

$x = \log_4(y+2)$

$y + 2 = 4^x$

$y = 4^x - 2$

$y^{-1} = 4^x - 2$

1.) $f(x) = -2x + 1$

2.) $y = \sqrt{x+1}$

3.) $f(x) = 4^x$

4.) $y = \dfrac{2x+1}{3}$

5.) $y = \log_3(x-1)$

6.) $y = 4^{x+2}$

7.) $y = \log_2(x+2)$

8.) $y = \sqrt[3]{x+3}$

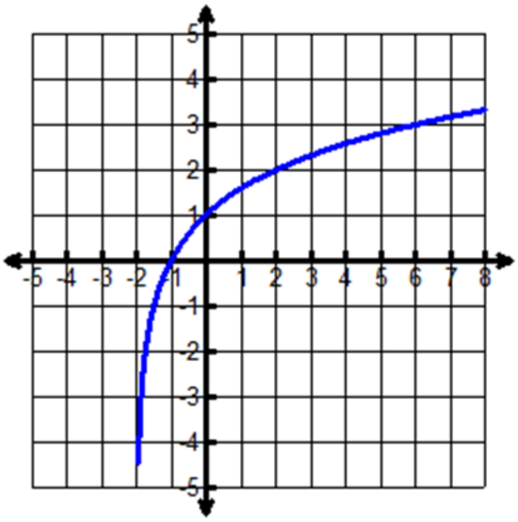
To find the inverse graphically:

1) Make a table of values for the function.

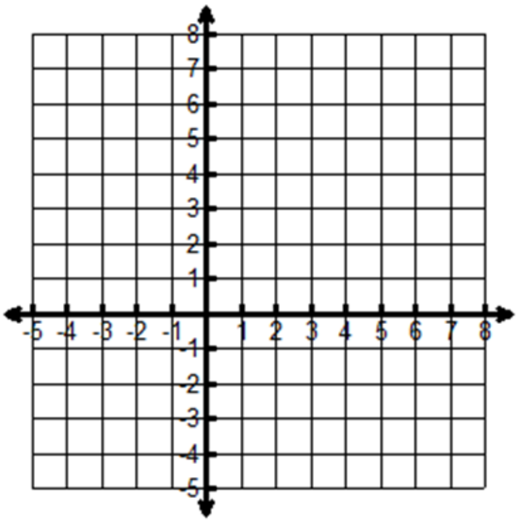
2) Make a new TOV where the domain and the range switch places.

3) Graph

14)



15) INVERSE OF #5

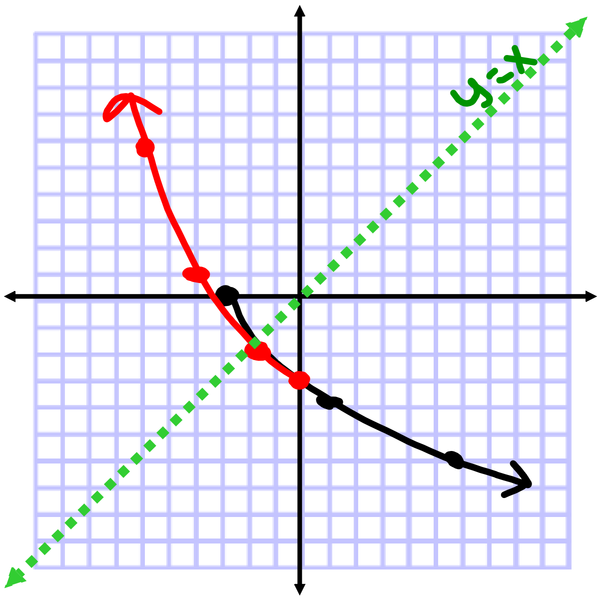


f

x	y
-3	0
-2	-2
1	-4
6	-6

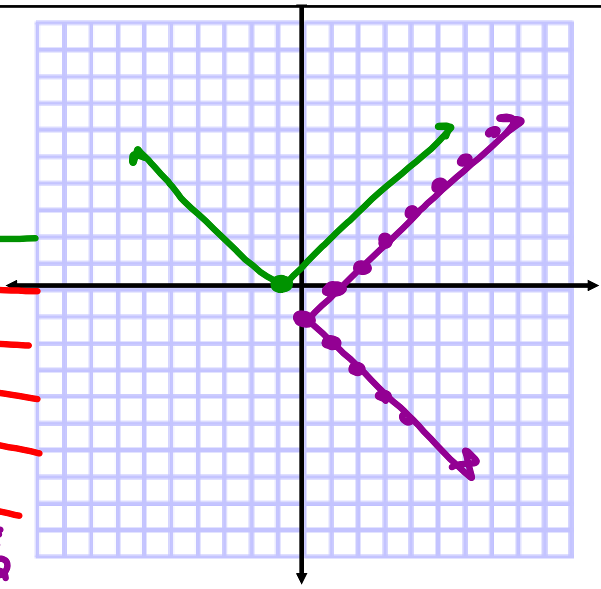
f^{-1}

x	y^{-1}
0	-3
-2	-2
-4	1
-6	6



x	y
-1	0
0	1
1	2
2	3
3	4
-2	1
-3	2
-4	3

x	y
0	-1
1	0
2	1
3	2
4	3
~~-2~~	
1	-2



Determining Whether f & g are inverses.

If :

$f(g(x)) = x$

$g(f(x)) = x$

Then f & g are inverses.

9) $f(x) = 3x - 6$ $g(x) = \frac{x+6}{3}$

$f(g(x)) = 3\left(\frac{x+6}{3}\right) - 6 = x + 6 - 6 = x$

$g(f(x)) = \frac{(3x-6)+6}{3} = \frac{3x}{3} = x$

Therefore f & g are inverses.

10) $f(x) = 4x^2 - 3$ $g(x) = \sqrt{\frac{x+3}{2}}$

$f(g(x)) = 4\left(\sqrt{\frac{x+3}{2}}\right)^2 - 3$

$= \cancel{4}\left(\frac{x+3}{\cancel{4}}\right) - 3 = x + 3 - 3 = x$

$g(f(x)) = \sqrt{\frac{(4x^2-3)+3}{2}} = \frac{\sqrt{4x^2}}{2} = \frac{2x}{2} = x$

Therefore f & g are inverses.

12) $f(x) = \log_3 x + 1$

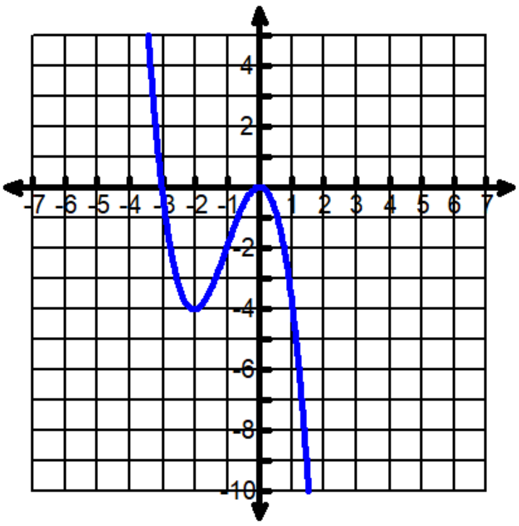
$g(x) = 3^{x-1}$

$f(g(x)) = \log_3(3^{x-1}) + 1 = x - 1 + 1 = x$

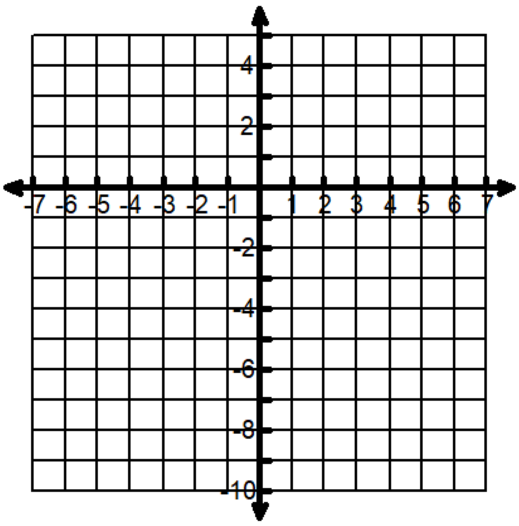
$g(f(x)) = 3^{(\log_3 x + 1) - 1} = 3^{\log_3 x} = x$

Therefore f + g are inverses.

6)



17) INVERSE OF # 6



Determine if f & g are inverses using $\boxed{\text{compositions!}}$

If ① $f(g(x)) = x$ and ② $g(f(x)) = x$

Then f & g are inverses.

a) $f(x) = 3x - 6$ $g(x) = \frac{x+6}{3}$

① $f(g(x)) = x$

$$f(g(x)) = 3\left(\frac{x+6}{3}\right) - 6$$
$$= x + 6 - 6$$
$$= x$$

$$g(f(x)) = \frac{(3x-6)+6}{3} = \frac{3x}{3} = x$$

Therefore, f & g are inverses.

10) $f(x) = 4x^2 - 3$ $\quad g(x) = \frac{\sqrt{x+3}}{2}$

$f(g(x)) = 4\left(\frac{\sqrt{x+3}}{2}\right)^2 - 3$

$= 4\left(\frac{x+3}{4}\right) - 3 = x + 3 - 3 = x$

$g(f(x)) = \frac{\sqrt{(4x^2-3)+3}}{2} = \frac{\sqrt{4x^2}}{2} = \frac{2x}{2} = x$

Therefore f + g are inverses.

11) $f(x) = \log_2 x$ $\quad g(x) = 2^x$

$f(g(x)) = \log_2(2^x) = x$

$g(f(x)) = 2^{(\log_2 x)} = x$

Therefore f + g are inverses.

12) $f(x) = \log_3 x + 1$ $g(x) = 3^{x-1}$

$f(g(x)) = \log_3(3^{x-1}) + 1 = x - 1 + 1 = x$

$g(f(x)) = 3^{(\log_3 x + 1) - 1} = 3^{\log_3 x} = x$

Therefore, f + g are inverses.

10) $f(x) = 4x^2 - 3$ $g(x) = \frac{\sqrt{x+3}}{2}$

13) $f(x) = \log_4 x - 2$ $g(x) = 4^x$