

Polynomial Inequalities

Steps to Solving Polynomial Inequalities:

1. Place the inequality in standard form.
2. Factor the polynomial completely and solve.
3. Use the zeros (do the zeros bounce or cross) of the polynomial and the end behavior to sketch the polynomial.
  - Greater than- Above the x-axis
  - Less than- Below the x-axis
4. Highlight the portion of the graph that makes the inequality true.
  - Greater than- Above the x-axis
  - Less than- Below the x-axis
5. Write the answer in interval notation.

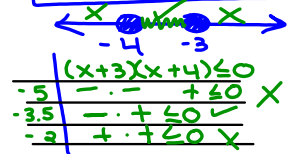
1)  $x^2 + 7x \leq -12$

Find Critical values (CV)

$x^2 + 7x = -12$   
 $x^2 + 7x + 12 = 0$   
 $(x+3)(x+4) = 0$   
 $x+3=0 \quad x+4=0$   
 $x=-3 \quad x=-4$

Answer:  $[-4, -3]$

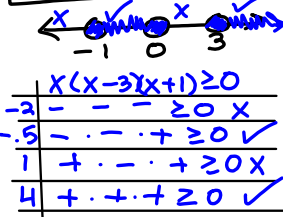
Real # & Line



2)  $x^3 - 2x^2 - 3x \geq 0$

RNL

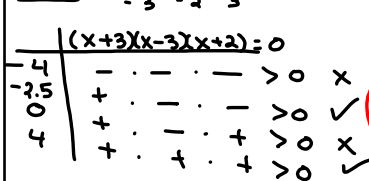
CV  
 $x^3 - 2x^2 - 3x = 0$   
 $x(x^2 - 2x - 3) = 0$   
 $x(x-3)(x+1) = 0$   
 $x=0 \quad x-3=0 \quad x+1=0$   
 $x=3 \quad x=-1$



Answer:  $[-1, 0] \cup [3, \infty)$

3.  $x^3 + 2x^2 - 9x - 18 \leq 0$

RNL



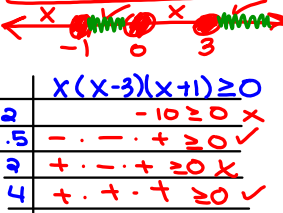
$x^3 + 2x^2 - 9x - 18 = 0$   
 $x^2(x+2) + (-9)(x+2) = 0$   
 $(x^2 - 9)(x+2) = 0$   
 $(x+3)(x-3)(x+2) = 0$   
 $x+3=0 \quad x-3=0 \quad x+2=0$   
 $x=-3 \quad x=3 \quad x=-2$   
 $x^2 - 9 = 0$   
 $x^2 = 9$   
 $x = \pm 3$

Answer:  $(-3, -2) \cup (3, \infty)$

2)  $x^3 - 2x^2 - 3x \geq 0$

Real # Line

Find CV  
 $x^3 - 2x^2 - 3x = 0$   
 $x(x^2 - 2x - 3) = 0$   
 $x(x-3)(x+1) = 0$   
 $x=0 \quad x-3=0 \quad x+1=0$   
 $x=3 \quad x=-1$

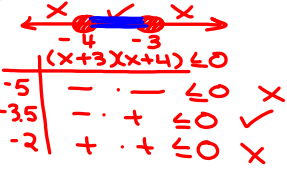


Answer:  $[-1, 0] \cup [3, \infty)$

$x^2 + 7x \leq -12$

Find CV


Real Number Line



Answer  $[-4, -3]$

4.  $(x-1)^2(x-11) \geq 0$

CV:  $x-1=0$     $x-11=0$   
 $x=1$     $x=11$

RNL: 

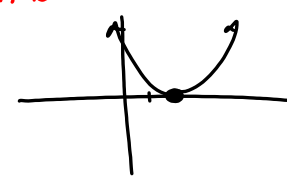
Answer:  $[1] \cup [11, \infty)$

	$(x-1)^2$	$(x-11)$	$\geq 0$	
0	+	-	< 0	x
5	+	-	< 0	x
15	+	+	> 0	✓

$(x-2)^2 > 0$

CV:  $x-2=0$   
 $x=2$

Answer:  $(-\infty, 2) \cup (2, \infty)$



	$(x-2)^2$	$> 0$
0	+	> 0 ✓
4	+	> 0 ✓

$(x-2)^2 < 0$

$(x-2)^2 \geq 0$


$(x-2)^2 \leq 0$

Now you try a couple ☺

$x^3 + 4x^2 - 16x - 64 \leq 64$

CV:  $x^3 + 4x^2 - 16x - 64 = 0$   
 $x^2(x+4) + (-16)(x+4) = 0$   
 $x^2 - 16 = 0$     $x+4 = 0$   
 $x^2 = 16$     $x = -4$   
 $x = \pm 4$

Answer:  $(-\infty, 4]$



	$(x+4)^2$	$(x-4)$	$\leq 0$
5	+	-	< 0 ✓
0	+	-	< 0 ✓
5	+	+	> 0 ✗

	$(x+4)(x-4)(x+4) \leq 0$
5	-
0	+
5	+

$(x-3)(x+1)^2(x+4) \geq 0$   
 CV:  $x = 3, -1, -4$   
 RNL

**Answer:**  
 $(-\infty, -4] \cup [-1, 3] \cup [3, \infty)$

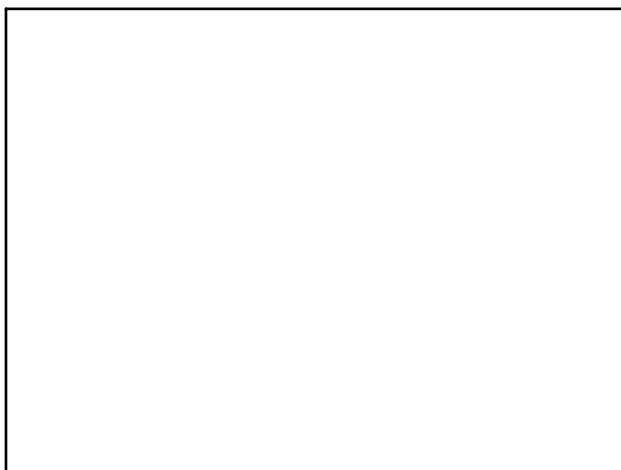
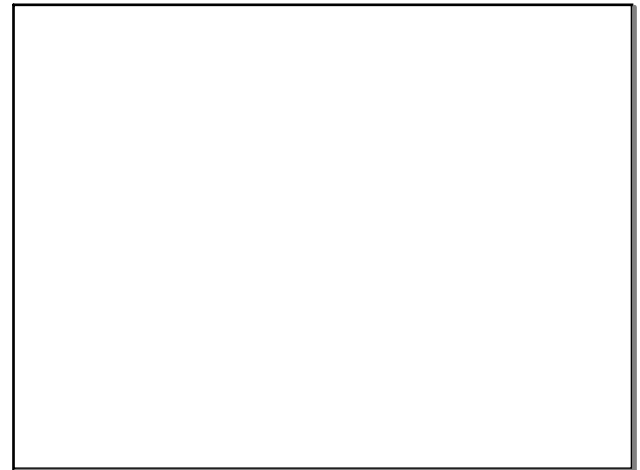
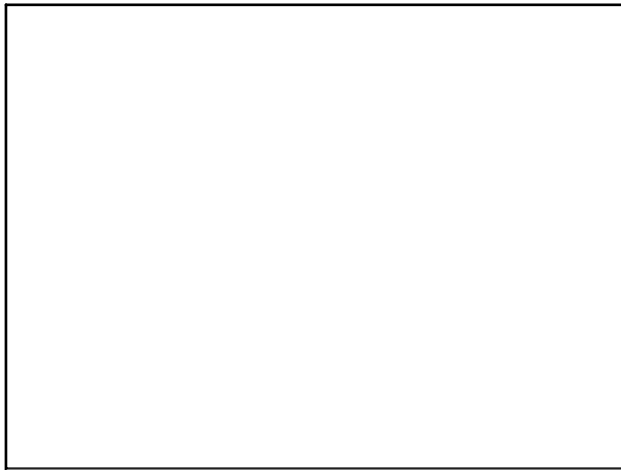
	$(x-3)(x+1)^2(x+4) \geq 0$	
$-\infty$	-	+
$-4$	+	-
$-1$	-	+
$3$	+	+
$\infty$	+	+

**Try:**  
 $(x+4)(x-2)(x-5) < 0$   
 CV:  $x = -4, 2, 5$   
~~RNL~~

**Answer:**  $(-\infty, -4) \cup (2, 5)$

$x$	$(x+4)(x-2)(x-5) < 0$
$-\infty$	-
$-4$	+
$2$	-
$5$	+
$\infty$	-

LC:  $x \cdot x \cdot x = x^3$  yint:  $4 \cdot (-2) \cdot (-5) = 40$



1.  $3x^2 - 5x - 12 < 0$

$$2. x^2 + 20x + 100 \geq 0$$

$$3. x^2 + x - 42 \geq 0$$

$$4. x^3 - 4x \geq 0$$

$$5. x^3 - 3x^2 - x + 3 < 0$$

$$6. 3x^2 + 5x - 2 > 0$$

$$7. x^2 - 9 \leq 0$$

8.  $4x^2 - 25 > 0$

9.  $3x^2 - 6x \geq 0$

10.  $2x^2 + 11x + 5 < 0$

11.  $x^3 + x^2 - 20x < 0$

12.  $(x+4)^2(x-4)(x-3) < 0$

13.  $(x-3)(x+1)^2(x+4) \geq 0$

$$14. x^3 - 5x^2 \leq 24x$$

$$15. x^3 - x^2 - 16x + 16 > 0$$

$$16. x^3 + 2x^2 - 4x - 8 \leq 0$$