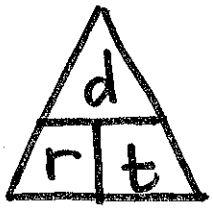
Word Problems



Distance $d = rt$

time $t = \frac{d}{r}$

Rate $r = \frac{d}{t}$

$$R_{TOGETHER} = R_1 + R_2$$

$$t_{total} = t_1 + t_2$$

* You may have to solve a system. *

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(1)

Boat rate	~~down~~ Upstream	Upstream
20	$20 + c$	$20 - c$
distance	3	2
	t_1	t_2

$$t_1 = t_2$$

$$\frac{3}{20+c} = \frac{2}{20-c}$$

{ $\frac{\text{distance}}{\text{rate}}$ = time }

$$60 - 3c = 40 + 2c$$

$$20 = 5c$$

$$\boxed{c = 4 \text{ mph}}$$

② Distance 720 720

Rates $x + 30$ $x - 30$

time total time = 10

* $t_{total} = t_1 + t_2$ *

* $t = \frac{\text{distance}}{\text{rate}}$ *

$$t_{total} = t_1 + t_2$$

CD $$\left(\frac{10}{1} = \frac{720}{x+30} + \frac{720}{x-30}\right)^{(x+30)(x-30)}$$

distrib. $10(x^2 - 900) = 720(x-30) + 720(x+30)$

$10x^2 - 9000 = 720x - 21600 + 720x + 21600$

set = 0 $10x^2 - 1440x - 9000 = 0$

Factor $x^2 - 144x - 900 = 0$

$(x - 150)(x + 6) = 0$

$x = 150$ $x = -6$

Rate of airplane = 150 mph

→ ~~150~~ going (with wind)

→ 120 coming back (against)

3)

	Bike	Driving
Distance	75	165
Rate	r	$r+30$
"same time" $\Rightarrow$	t_1	t_2

* Find r *

$t_1 = t_2$

time = distance / rate :

$$\frac{75}{r} = \frac{165}{r+30}$$

$$75(r+30) = 165r$$

$$75r + 2250 = 165r$$

$$2250 = 90r$$

$$r = 25 \text{ mph}$$

bike speed = 25 mph

driving speed = 55 mph

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4)

	Portion A	Portion B
Distance	540	360
time	$t+1$	t
Rates	r_1	r_2

rate = distance / time

* $r_1 = r_2$ *

* Find t *

$r_1 = r_2$

$$\frac{540}{t+1} = \frac{360}{t}$$

$$540t = 360t + 360$$

$$180t = 360 \rightarrow t = 2 \text{ hours}$$

Portion B: 2 h
Portion A: 3 h
Total : 5 h

5)

	Inlet	Outlet	Total
Distance	1	-1	1
time	10h	30h	t

Rate = $\frac{\text{Distance}}{\text{time}}$

$R_1 - R_2 = R_{Together}$

$\left(\frac{1}{10}\cdot\frac{30t}{1} - \frac{1}{30}\cdot\frac{30t}{1} = \frac{1}{t}\cdot\frac{30t}{1}\right)$ CD: 30t

$3t - t = 30$

$2t = 30$

$\boxed{t = 15 \text{ hours}}$

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6)

	Upstream	Downstream	Total
Distance	6	6	= 12
Rate	r-2	r+2	
time	t_1	t_2	4

$t = \frac{\text{distance}}{\text{rate}}$

$t_1 + t_2 = t_{total}$

$\left(\frac{6}{r-2} + \frac{6}{r+2} = 4\right)(r-2)(r+2)$

$6(r+2) + 6(r-2) = 4(r^2-4)$

$6r + 12 + 6r - 12 = 4r^2 - 16$

$0 = 4r^2 - 12r - 16$

$0 = r^2 - 3r - 4$

$0 = (r+1)(r-4)$

~~$r = -1$~~

$r = 4$

Rate in Still water 4mph

train	bus
350	350
$t-2$	t
$r+20$	r

$t = \frac{\text{distance}}{\text{rate}}$

$t = \frac{350}{r}$

$d_1 = d_2$ $\quad$ $d = r \cdot t$

d_1 $\quad$ d_2

$(r+20)(t-2) = 350$

$(r+20)\left(\frac{350}{r}-2\right) = 350$

$(r+20)(350-2r) = 350r$

$350r - 2r^2 + 7000 - 40r = 350r$

$0 = 2r^2 + 40r - 7000$

$0 = r^2 + 20r - 3500$

$0 = (r+70)(r-50)$

$r = -70$ $\quad$ $\boxed{r = 50}$

So, the train's speed is 70 mph. The bus's speed is 50 mph.

7)

	Roadcrew 1	Roadcrew 2	together
Distance	1	1	1
Time	15 h	10 h	t
Rate	R_1	R_2	$R_{together}$

rate = $\frac{\text{distance}}{\text{time}}$

$r_1 + r_2 = r_{together}$

$\left(\frac{1}{15} + \frac{1}{10} = \frac{1}{t}\right) \cdot 30t$

$2t + 3t = 30$

$5t = 30$

$t = 6$

It would take 6 hours.

9)

8)

	E	A	Total
Distance	1	1	1
time	20	t	12

Rate = $\frac{\text{Distance}}{\text{time}}$

$\text{Rate}_E + \text{Rate}_A = R_{\text{Total}}$

$\left(\frac{1}{20} + \frac{1}{t} = \frac{1}{12}\right) \cdot 60t$

CD: 60t

$3t + 60 = 5t$

$60 = 2t$

$\boxed{t = 30 \text{ hours}}$

$D_1: 2 \cdot 2 \cdot 5$

$D_2: E$

$D_3: 2 \cdot 2 \cdot 3$

CD: $2 \cdot 2 \cdot 5 \cdot 3t$

CD = 60t

Puzzle

	Distance	time	Rates
Gammill	1	5 hours	1/5
Dill	1	t	1/t
together	1	3 hours	1/3

How long does it take Dill to put together the puzzle

Rate = $\frac{\text{distance}}{\text{time}}$

$R_G + R_D = R_T$ } CD: 15t

$\frac{1}{5} + \frac{1}{t} = \frac{1}{3}$

$3t + 15 = 5t$

$15 = 2t$

$\boxed{t = 7.5 \text{ hours}}$

9)

	Experienced	Helper	Together
Distance	1	1	1
Time	$\boxed{t}$	$2t$	4
Rate	$2r$	r	$1/4$

Find t

That means that

Experienced → 1 house in 6 hours

Helper → 1 house in 12 hours

Together → 1 house in 4 hours.

$$r_1 + r_2 = r_{together}$$

$$\frac{1}{t} + \frac{1}{2t} = \frac{1}{4}$$

$$4 + 2 = t$$

$$6 = t$$

$\boxed{t = 6 \text{ hours}}$

only

$r_1 + r_2 = r_{To}$ $\quad t = \frac{1}{2r} = \frac{1}{2(1/12)} =$ (6 hours)

$2r + r = 1/4$

$3r = \frac{1}{4}$

$r = \frac{1}{12}$

$t_1 + t_2 = 4$

$t + 2t = 4$

$3t = 4$

$t = 4/3$

$\frac{4}{8} = \frac{8}{3} = \frac{12}{9}$

$\frac{4 \text{ houses}}{3 \text{ hours}} = \frac{1 \text{ house}}{x}$

$t_1 + t_2 = 4$

$\frac{1}{2r} + 2\left(\frac{1}{r}\right) = 4$

$\left(\frac{1}{2r} + \frac{2}{r} = \frac{4}{1}\right) 2r$

$1 + 4 = 8r$

$r = 5/8$

$r = \frac{5 \text{houses}}{8 \text{hours}}$.

$t = \frac{1}{2r}$

$t_1 = \frac{1}{2(5/8)} = \frac{8}{10} = \frac{4}{5}$

$t_2 = \frac{8}{5}$

$2t = \frac{1}{r}$

$t = \frac{1}{}$

1 house: $\frac{1}{4}t + \frac{1}{2}t = 1$

: $\frac{1}{4} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{4}{5} = 1 \text{ house}$

Not True.

$\frac{1}{2}t + t = 4$

$\frac{3}{2}t = 4$

$3t = 8$

$t = 8/3$

$\frac{1 \text{ ho}}{8 \text{ hr}} = \frac{1 \text{ house}}{t}$

$5t = 8$

$t = 8/5$

$10t = 8$

$t = 8/10$

11)

	V	C	Together
Distance	1	1	1
time	$t-5$	t	6

$$R_1 + R_2 = R_{Together}$$

$$\frac{1}{t-5} + \frac{1}{t} = \frac{1}{6} \quad \{CD: 6t(t-5)$$

$$6t + 6(t-5) = t(t-5)$$

$$6t + 6t - 30 = t^2 - 5t$$

$$0 = t^2 - 17t + 30$$

$$0 = (t-2)(t-15)$$

$t = \cancel{2}$ $\quad \boxed{t = 15}$

Virginia takes 10 hours
Carl takes 15 hours

1 30
2 15
3 10
5 6

10)

	Scanner 1	Scanner 2	Total
Distance	1	1	1
time	$t+12$	t	8
Rate	$\frac{1}{t+12}$	$\frac{1}{t}$	$\frac{1}{8}$

*When two things are happening at once

$t_1 + t_2 \neq$ total*

$R_1 + R_2 = R_{Total}$

$$R_1 + R_2 = R_{Together}$$

$$\frac{1}{t+12} + \frac{1}{t} = \frac{1}{8}$$

$$8t + 8(t+12) = t(t+12)$$

$$8t + 8t + 96 = t^2 + 12t$$

$$0 = t^2 - 4t - 96$$

$$0 = (t-12)(t+8)$$

$t = 12$

$\cancel{t = -8}$

Scanner 1 24 min

Scanner 2 12 min

(12)

	Model 1	Model 2	Total
distance	1	1	1
time	t_1	t_2	8
rate	$3r$	r	$1/8$

What's t_2?

$3r + r = \frac{1}{8}$

$4r = \frac{1}{8}$

$r = \frac{1}{32}$

$t = \frac{d}{r} = \frac{1}{1/32} = 32$ mins

(13)

	Landscaper	Assistant	Total
distance	1	1	1
time	12	t	8
rate	r_1	r_2	$\frac{1}{8}$

$r_1 + r_2 = r_{together}$

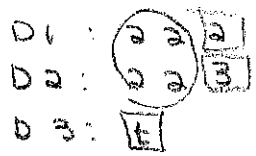
$\frac{1}{12} + \frac{1}{t} = \frac{1}{8}$

$2t + 24 = 3t$

$\boxed{t = 24 \text{ hours}}$

14)

	Water In	Water Out	Total
distance	1	1	1
time	8	12	t
rate	r_1	r_2	$\frac{1}{t}$



$\frac{1}{8} + \frac{1}{12} = \frac{1}{t}$

$3t + 2t = 24$

$5t = 24$

$t = 4.8$ hours

It will take 24 minutes to fill the tub.

15)

	Printer 1	Printer 2	Total
Distance	1	1	1
time	60 min	90 min	t
rate	r_1	r_2	$r_{together}$

$\frac{1}{1} + \frac{1}{1.5} = \frac{1}{t}$

$1.5t + t = 1.5$

$2.5t = 1.5$

$25t = 15$

$t = 15/25 = 3/5 = 36$ minutes

16)

	Private	Commercial	Total
distance	720	720	$d_1 = d_2$
time	t	$t-2$	$t(r-180) = r(t-2)$
rate	$r-180$	r	$\frac{720}{r-180} \cdot r-180 = r\left(\frac{720}{r-180} - 2\right)$

$\frac{720}{r-180} = t$

$720r - 18(720) = 720r - 2r^2 + 360r$

$2r^2 - 360r - 180(720) = 0$

$(r-18)720 = 720r + 2r(r-180)$

$r^2 - 180r - 6480 = 0$

$r \approx 210$ mph

Private 30 mph?

16)

	Private	Commerical
distance	720	720
rate	$r-180$	r
time	$t+2$	t

$$\text{time} = \frac{\text{distance}}{\text{rate}} = \boxed{\frac{720}{r}}$$

$d_1 = d_2 \rightarrow \underbrace{(t+2)(r-180)}_{d_1} = \underbrace{720}_{d_2}$

Substitute $\left(\frac{720}{r}+2\right)(r-180)=720$

$\cdot r$ $(720+2r)(r-180)=720r$

multiply $720r - 720(180) + 2r^2 - 360r = 720r$

CLT $2r^2 - 360r - 129600 = 0$

$\div 2$ $r^2 - 180r - 64800 = 0$

Factor $(r+180)(r-360)=0$

Solve $r=-180 \quad r=360.$

So-
The rate of the private plane is $360-180=\boxed{180}$

The rate of the commerial is $\boxed{360}$

17)

	Day 1	Day 2
distance	240	144
time	$t+2$	t
rate	r_1	r_2

$r_1 = r_2$

$\text{rate} = \frac{\text{distance}}{\text{time}}$

Since $r_1 = r_2$

$$\frac{240}{t+2} = \frac{144}{t}$$

$240t = 144t + 288$

$96t = 288$

$t = 3$

Day 1 = 5 hours

Day 2 = 3 hours

	Passenger	Freight
distance	275	225
time	t_1	t_2
rate	$r+10$	r

$t_1 = t_2$

$t = \frac{\text{distance}}{\text{rate}}$

$t_1 = t_2 \Rightarrow \frac{275}{r+10} = \frac{225}{r} \rightarrow 275r = 225r + 2250$

$50r = 2250$

$r = 45$

Passenger Rate = 55 mph

Freight Rate = 45 mph

19) Plane 300

	Plane with tailwind	Plane against tailwind
distance	85	65
time	t_1	t_2
rate	$300 + \text{tailwind}$	$300 - \text{tailwind}$

* $t_1 = t_2$
* $t = \frac{d}{r}$

$t_1 = t_2$

$\frac{85}{300+x} = \frac{65}{300-x}$

$85(300 - x) = 65(300 + x)$

$25500 - 85x = 19500 + 65x$

$6000 = 150x$

$\boxed{x = 40}$

→ The tailwind is 40 mph.